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ON THE TEMPERATURE IN A NON-HOMOGENEOUS BAR IN ASSOCIATION WITH $\overline{H}$ --FUNCTION

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ABSTRACT

We employ the $\overline{H}$ -function to obtain the formal solution of the partial differential equation:

$$\frac{\partial v}{\partial t} = \lambda \frac{\partial}{\partial u}\left[(1-u^2)\frac{\partial v}{\partial u}\right]$$

Related to a problem of heat conduction by making use of the integral and orthogonality property of Jacobi polynomials. The result generalizes a number of known particular cases on specialization of the parameters. (2000 Mathematics subject classification: 33c99)

KEYWORDS: $\overline{H}$ -function, H -function, Jacobi polynomials, Partial differential equation, Orthogonality property.

INTRODUCTION

The $\overline{H}$ -function occurring in the paper will be defined and represented by Buschman and Srivastava [1] as follows:

$$\overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N}\left[z \left| \begin{matrix} (a_j;\alpha_j;A_j)_{1,N},(a_j;\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right.\right]$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \overline{\phi}(\xi) z^{\xi} d\xi \qquad (1.1)$$

where

$$\overline{\phi}(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M+1}^{Q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \qquad (1.2)$$

Which contains fractional powers of the gamma functions. Here, and throughout the paper $a_j(j=1,...,p)$ and $b_j(j=1,...,Q)$ are complex parameters, $\alpha_j \geq 0(j=1,...,P), \beta_j \geq 0(j=1,...,Q)$ (not all zero simultaneously) and exponents $A_j(j=1,...,N)$ and $B_j(j=N+1,...,Q)$ can take on non integer values.

The following sufficient condition for the absolute convergence of the defining integral for the $\overline{H}$ -function given by equation (1.1) have been given by (Buschman and Srivastava[1]).

$$\Omega \equiv \sum_{j=1}^{M}|\beta_j| + \sum_{j=1}^{N}|A_j\alpha_j| - \sum_{j=M+1}^{Q}|\beta_j B_j| - \sum_{j=N+1}^{P}|\alpha_j| > 0 \qquad (1.3)$$

and $|\arg(z)| < \frac{1}{2}\pi\Omega$ (1.4)

The behavior of the $\overline{H}$ -function for small values of $|z|$ follows easily from a result recently given by (Rathie [5],p.306,eq.(6.9)).

We have

$$\overline{H}^{M,N}_{P,Q}[z] = 0\left(|z|^{\gamma}\right), \gamma = \min_{1\le j\le N}\left[\mathrm{Re}\left(b_j / \beta_j\right)\right], |z| \to 0 \quad (1.5)$$

If we take $A_j = 1(j = 1,...,N)$, $B_j = 1(j = M+1,...,Q)$ in (1.1), the function $\overline{H}^{M,N}_{P,Q}$ reduces to the Fox's H-function [3].

We shall use the following notation:

$A^* = \left(a_j, \alpha_j; A_j\right)_{1,N}, \left(a_j, \alpha_j\right)_{N+1,P}$ and $B^* = \left(b_j, \beta_j\right)_{1,M}, \left(b_j, \beta_j; B_j\right)_{M+1,Q}$

We require the following result:

$$\int_{-1}^{1}(1-u)^{e}(1+u)^{\beta}P_r^{(\alpha,\beta)}(u)\,{}_PF_Q\left[{}^{A_P}_{B_Q}; d\left(\frac{1-u}{2}\right)^{g}\right]$$

$$\overline{H}^{m,n}_{p,q}\left[z\left(\frac{1-u}{2}\right)^{h}\Big|^{A^*}_{B^*}\right]du =$$

$$\sum_{c=0}^{\infty}\frac{(A_P;c)d^{c}(-1)^{r}2^{\alpha+\beta+1}\Gamma(1+\beta+r)}{(B_Q;c)\ c!r!}$$

$$\overline{H}^{m,n}_{p+2,q+2}\left[z\Big|^{(a_j;\alpha_j;A_j)_{1,n},(-e-gc,h),(\alpha-e-gc,h),(a_j;\alpha_j)_{n+1,p}}_{B^*,(\alpha+r-gc-e,h;1),(-1-\beta-r-e-gc,h;1)}\right] \quad (1.6)$$

Where

$$\mathrm{Re}\left(e + h\frac{b_j}{\beta_j}\right) > -1;\ i = 1,2,...,m$$

$$\Omega > 0, |\arg z| < \frac{1}{2}\Omega\pi, h > 0, P \le Q(Q+1 and |d| < 1), g > 0.$$

Th eorthogonality property of the Jacobi polynomials:

$$\int_{-1}^{1}(1-u)^{\alpha}(1+u)^{\beta}P_r^{(\alpha,\beta)}(u)P_s^{(\alpha,\beta)}(u)du = h_r\delta_{rs} \quad (1.7)$$

Where

$\mathrm{Re}(\alpha) > -1, \mathrm{Re}(\beta) > -1$ and

$$h_r = \frac{2^{\alpha+\beta+1}\Gamma(\alpha+r+1)\Gamma(\beta+r+1)}{r!(\alpha+\beta+1+2r)\Gamma(\alpha+\beta+1+r)}$$

And δ_{rs} is Kronecker delta function, defined as:

$$\delta_{rs} = {}^{0,\, if\ r\neq s}_{1,\, if\ r=s}$$

HEAT CONDUCTION AND $\overline{H}$ -FUNCTION

Our problem is to find a function $v(u,t)$ representing the tempreture in a non-homogeneous bar with ends at $u = \pm 1$ in which the thermal conductivity is proportional to $(1-u^2)$ and if the ateral surface of the bar is insulated, it satisfies the partial differential equation of heat conduction

$$\frac{\partial v}{\partial t} = \lambda \frac{\partial}{\partial u}\left[(1-u^2)\frac{\partial v}{\partial u}\right] \qquad (2.1)$$

Where λ is a constant, provided the thermal coefficent is constant.

The boundaru conditions of the problem are that both ends of a bar at $u = \pm 1$ are also insulated because the conductivity vanishes there, and the initial conditions:

$$v(u,0) = f(u), \ \ -1 < u < 1 \qquad (2.2)$$

In view of (2.2), we consider

$$v = f(u) = (1-u)^e \, {}_P F_Q\left[{}^{A_P}_{B_Q}; d\left(\frac{1-u}{2}\right)^g \right]$$

$$\overline{H}^{m,n}_{p,q}\left[z\left(\frac{1-u}{2}\right)^h \Big|^{A^*}_{B^*} \right] du \qquad (2.3)$$

We may therefore, assume the solution of (2.1) in the form

$$v(u,t) = \sum_{w=0}^{\infty} R_w e^{-\lambda w(w+1)t} P_w^{(\alpha,\beta)}(u) \qquad (2.4)$$

When $t = 0$ in (2.4) and using (2.3), we have

$$f(u) = (1-u)^e \, {}_P F_Q\left[{}^{A_P}_{B_Q}; d\left(\frac{1-u}{2}\right)^g \right] \overline{H}^{m,n}_{p,q}\left[z\left(\frac{1-u}{2}\right)^h \Big|^{A^*}_{B^*} \right]$$

$$= \sum_{w=0}^{\infty} R_w P_w^{(\alpha,\beta)}(u) \qquad -1 < u < 1 \qquad (2.5)$$

Here $P_w^{(\alpha,\beta)}(u)$ is a Jacobi polynomial.

Equation (2.3) is valid since $f(u)$ is continuous in the closed interval $- \le u \le 1$ and has a piece-wise continuous derivative there, then $\alpha > -1, \beta . -1$, the Jacobi series associated with $f(u)$ converges uniformly to $f(u)$ in $-1 + \in \le u \le 1 - \in, \ 0 < \in < 1$.

Now multiplying both sides of (2.5) by $(1-u)^\alpha (1+u)^\beta P_r^{(\alpha,\beta)}(u); \alpha > -1, \beta > -1$ and integrating from -1 to 1, the use of (1.6) yields

$$A_r = h_r^{-1} \int_{-1}^{1} (1-u)^{e+\alpha} (1+u)^\beta P_r^{(\alpha,\beta)}(u) \, {}_P F_Q\left[{}^{A_P}_{B_Q}; d\left(\frac{1-u}{2}\right)^g \right]$$

$$\overline{H}^{m,n}_{p,q}\left[z\left(\frac{1-u}{2}\right)^h \Big|^{A^*}_{B^*} \right] du \qquad (2.6)$$

We freely apply (1.6) in (2.6) to obtain

$$A_r = \frac{(-1)^r 2^{\alpha+\beta+1}\Gamma(1+\alpha+\beta+r)\Gamma(1+\alpha+\beta+2r)}{\Gamma(1+\alpha+r)}$$

$$\sum_{e=0}^{\infty} \frac{(A_P;c)d^c}{(B_Q;c)e!} \overline{H}^{m,n}_{p+2,q+2}\left[z \Big|^{(a_j;\alpha_j;A_j)_{1,n},(-e-gc,h),(-\alpha-e-gc,h),(a_j;\alpha_j)_{n+1,p}}_{B^*,(-\alpha+r-gc-e,h;1),(-1-\alpha-\beta-r-e-gc,h;1)} \right] \qquad (2.7)$$

On substituting the value of R_w from (2.7) in (2.4), we arrive at the desired solution.

$$v(u,t) = 2^e \sum_{w,e=0}^{\infty} f(w) e^{-\lambda w(w+1)t}$$

$$\overline{H}^{m,n}_{p+2,q+2}\left[z \Big|^{(a_j;\alpha_j;A_j)_{1,n},(-e-gc,h),(-\alpha-e-gc,h),(a_j;\alpha_j)_{n+1,p}}_{B^*,(-\alpha+r-gc-e,h;1),(-1-\alpha-\beta-r-e-gc,h;1)} \right] \qquad (2.8)$$

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Where

$$f(w) = \frac{(-1)^w 2^{\alpha+\beta+1}\Gamma(1+\alpha+\beta+w)\Gamma(1+\alpha+\beta+2w)}{\Gamma(1+\alpha+w)} \frac{(A_p;c)d^c}{(B_Q;c)e!}$$

And the conditions of validity are:

$$\mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0, \Omega > 0, |\arg z| < \frac{1}{2}\Omega\pi, h > 0, P \le Q(Q+1 and |d| < 1), g > 0 .$$

SPECIAL CASE

If we take $A_j = 1\left(j = 1,...,N\right)$, $B_j = 1\left(j = M+1,...,Q\right)$, we get the result due to Chaurasia [3].

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